

Logic and Emotion: Whitehead's Category of the Ultimate

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Process and Reality

- *An Essay in Cosmology*
 - subtitle given by Alfred North Whitehead to his celebrated Gifford lectures:
 - *PROCESS & REALITY (PR)* at Edinburgh in the session of 1927-28.
 - Published as Whitehead, Alfred North, *Process and Reality: An Essay in Cosmology*, corrected edition, edd D R Griffin, D W. Sherburne, New York, The Free Press (1978).
 - <https://archive.org/details/AlfredNorthWhiteheadProcessAndReality>
 - Two conflicting original editions, poor proof reading, inconsistencies, resolved as far as possible in corrected edition.

Process and Reality 2

- His cosmology is developed in terms of a Categorical Scheme
 - Philosophy of Organism.
 - Described as his speculative philosophy.
 - The foundation of his whole scheme of cosmology is the *Category of the Ultimate*.
 - A category in process terms is a typing and this fundamental category of his “expresses the general principle presupposed in the three more special categories”

Earlier Work

- Alfred North Whitehead's Process and Reality
 - Alex Scott
 - <https://www.angelfire.com/md2/timewarp/whitehead.html>
- Robert Rosen's Life Itself
 - Explores categorical relations in biological systems
- Heather & Rossiter:
 - Heather, M, & Rossiter, N, The Topos of Category Theory and Reality, Proc XVII International Conference on the Science and Quality of Life, June 29 - July 2 2013, Vilnius, Lithuania, edited Romuald Brazis, Studium Vilnense A 11 204-213 (2014).
 - Heather, Michael, & Rossiter, Nick, The Process Category of Reality, ANPA 31, Cambridge 224-262 (2011).
 - Heather, Michael, & Rossiter, Nick, Adjoint Typing: Is it Whitehead's Category of the Ultimate? 7th International Whitehead Conference January 5-9, Bangalore, Karnataka, India pp.72-74 (2009).
 - Heather, Michael, & Rossiter, Nick, Process Category Theory, Salzburg International Whitehead Conference, University of Salzburg 3-6 July (2006).

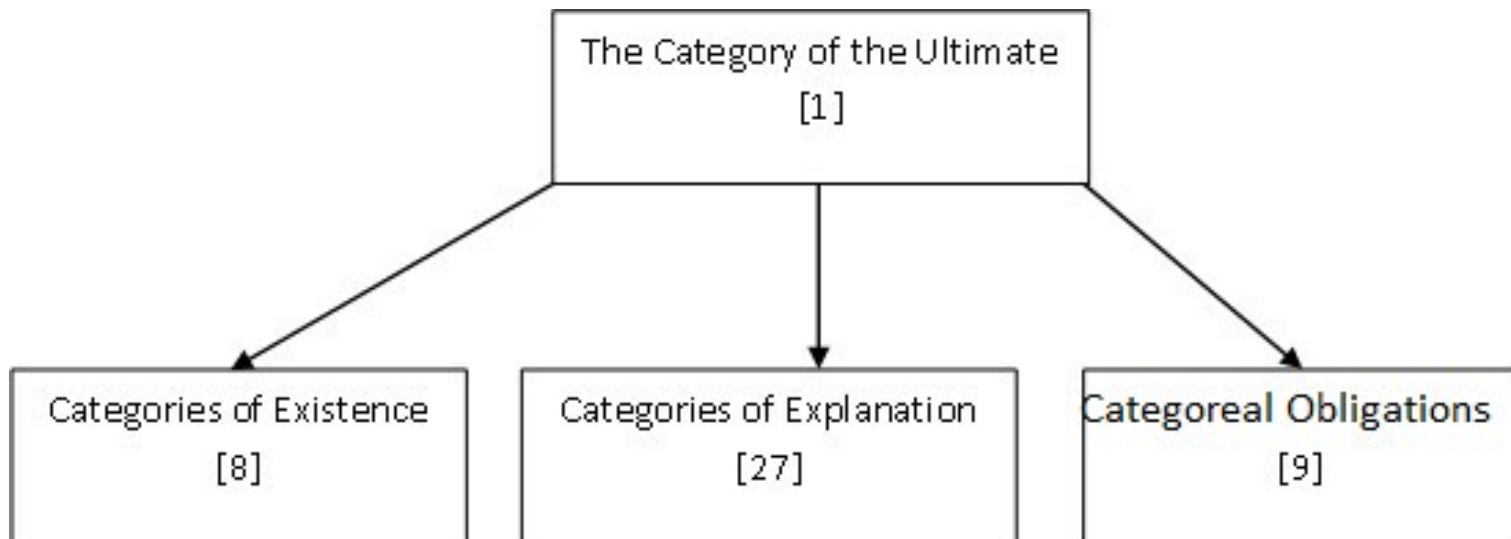
Rationale

- Whitehead's writing style is dense, involving many of his own terms
- Closer look is warranted on his actual text:
 - Increased understanding of category theory
 - Focus on concepts ANW considered important
- Desire to extend formalisms into biological sciences
 - ANW unifies the physical and biological

The Categoreal Scheme

- The special categories are:
 - Category of the Ultimate
 - Categories of Existence
 - Categories of Explanation
 - Categoreal Obligations
- These special categories are composed of
 - eight categories of existence
 - twenty seven categories of explanation
 - nine categoreal obligations
- “The Category of the Ultimate expresses the general principle presupposed in the three more special categories” p.21
- The whole of PR rests on this categoreal scheme

At first sight there seems to be a hierarchical typing relationship among these categories which might look like this:



Numbers are count of categories of each type

What Whitehead does not say

- However, Whitehead does not provide such a diagram in PR.
- Nor does he state that there is a hierarchical relationship.
- Whitehead does not even explain what he means by the term 'category'.
 - It seems it is defined by the Category of the Ultimate itself and therefore is self-referencing.
 - May be Aristotelian p.30
 - “ This Category of the Ultimate replaces Aristotle's category of 'primary substance’ “ p.21 (classify, predicate)
- If he has no difficulty with a category being a member of itself,
 - then a category is not to be identified with a set,
 - the concept earlier promoted in his other *magnum opus* (co-authored with Bertrand Russell) the *PRINCIPIA MATHEMATICA (PM)*.

Speculative Nature

- Whitehead seems very conscious of the speculative nature of his philosophy at this stage.
- The whole of Part I of *PR* is headed 'a Speculative Scheme'.
- It is speculative perhaps because at the time
 - he was giving the Gifford lectures and
 - for the remaining 20 years of his life
 - there was no formal presentation available for *PR*
 - as he and Russell were able to provide for *PM*.

One-Substance Approach

- Descartes (and other philosophers) considered there were two substances
 - Material body
 - Mental (mind, thought)
- Whitehead abandoned this approach, considering there was just one substance
 - So unifying mind and matter as a single process
- Whitehead describes his system of speculative philosophy as a ‘philosophy of organism’ or ‘organic philosophy,’ since he views reality as consisting of interrelated and mutually dependent parts that are involved in sustaining vital processes

Whitehead PR p.19

“With the purpose of obtaining a one-substance cosmology, 'prehensions' are a generalization from Descartes' mental 'cogitations,' and from Locke's 'ideas,' to express the most concrete mode of analysis applicable to every grade of individual actuality. Descartes and Locke maintained a two-substance ontology—Descartes explicitly, Locke by implication. Descartes, the mathematical physicist, emphasized his account of corporeal substance; and Locke, the physician and the sociologist, confined himself to an account of mental substance. “

Perhaps no longer a speculative philosophy

- New formal techniques are available
- So this speculative state of affairs may no longer hold.
- There is now a formal theory of categories only just beginning at the time of Whitehead's death in 1947 but now maturing

CT Foreshadowed by ANW

- Category Theory is a theory foreshadowed in Whitehead's *Category of the Ultimate*
 - quite comprehensively in the sense of his preface to *PR* at p. vi:
- Motivation for a complete cosmology
 - to construct a system of ideas which bring the aesthetic, moral and religious interest into relation with those concepts of the world which have their origin in natural science.
 - [Whitehead *PR* Part I]

Problems with Hierarchy

- From the formal theory of categories
 - Can understand the need for interdependence between categories
 - Not achieved in a hierarchy
 - So Whitehead presumably dismissed the use of hierarchies.
- The relationship is more complex than the hierarchy
 - in the same way Russell used the phrase ‘ramified type-theory’ rather than ‘hierarchical type-theory’ although both words contain the sense of a tree.

Whitehead's Proposals PR p.20

“Actual entities involve each other by reason of their prehensions of each other. There are thus real individual facts of the togetherness of actual entities, which are real, individual, and particular, in the same sense in [30] which actual entities and the prehensions are real, individual, and particular. Any such particular fact of togetherness among actual entities is called a ‘nexus’ (plural form is written 'nexūs'). The ultimate facts of immediate actual experience are actual entities, prehensions, and nexus. All else is, for our experience, derivative abstraction.”

Entities

- Real (exist)
- Individual (atomic)
- Particular (singled out, identity)
- Can be joined together as a nexus (union of similar types, ordered society)
- “Every entity should be a specific instance of one category of existence” PR p.20 (classification)
- What is prehension?

Prehension

- An overloaded word
 - Grasping, seizing
 - An interaction of a subject with an event or entity which involves perception but not necessarily cognition
- “There are eight Categories of Existence (PR p.22):
 - (i) Actual Entities (also termed Actual Occasions), or Final Realities, or Res Verae [true thing].
 - (ii) Prehensions, or Concrete Facts of Relatedness.”

Prehension – some other mentions

PR p.19

“A prehension reproduces in itself the general characteristics of an actual entity: it is referent to an external world, and in this sense will be said to have a 'vector character'; it involves emotion, and purpose, and valuation, and causation. In fact, any characteristic of an actual entity is reproduced [29] in a prehension. It might have been a complete actuality; but, by reason of a certain incomplete partiality, a prehension is only a subordinate element in an actual entity.”

- Here a prehension describes an entity's characteristics
 - An example of adjointness

Prehension – PR p.23

“(xi) That every prehension consists of three factors: (a) the 'subject' which is prehending, namely, the actual entity in which that prehension is a concrete element; (b) the 'datum' which is prehended; (c) the 'subjective form' which is how that subject prehends that datum.”

- This is a data relationship, either X or $+$.
- A pullback (or pushout) category:
 - (c) is (a) $X_{(b)}D$ (D is a third entity)
- Adjoint if Locally Cartesian-closed

Prehension – General

- Whitehead describes prehensions as concrete modes of analysis of the world.
- To prehend something is to have a concrete idea or concept of that thing.
- However, prehension is not merely a mode of thinking:
 - A prehension is a process of appropriation of an element of an actual entity or of an element that is derived from an actual entity.
 - A prehension of an object or of an element of an object changes the internal constitution of the preheating subject.
 - Prehension is a process by which an actual entity, or preheating subject, becomes itself by appropriating elements from other actual entities.
 - The becoming of an actual entity occurs through a concrescence of prehensions.
 - Satisfaction is a final phase of concrescence (or the process of integration of feeling), in which prehensions are integrated into a concrete unity. A feeling is the integration of an actual entity or occasion into the internal constitution of a subject.

Prehension - General

- This is similar to adjointness (grasping, snap) with condescence being the free functor, creating a new emergent entity, with obligations as the underlying functor and satisfaction as the final integrity check.
- A general interpretation of prehension appears to be adjointness.

Prehension – further mentions

- p.35 “ There is a prevalent misconception that 'becoming' involves the notion of a unique seriality for its advance into novelty. This is the classic notion of 'time' which philosophy took over from common sense. [snap, not serial] “”
- p.52 “ The 'prehension' of one actual entity by another actual entity is the complete transaction, analysable into the objectification of the former entity as one of the data for the latter, and into the fully clothed feeling whereby the datum is absorbed into the subjective satisfaction —'clothed' with the various elements of its 'subjective form. “ [transaction = snap]

Adjointness Refresher

- For $F: \mathbf{L} \rightarrow \mathbf{R}$ and $G: \mathbf{R} \rightarrow \mathbf{L}$
 - F, G are Functors
 - \mathbf{L}, \mathbf{R} are Categories
 - $F \dashv G$ that is F is left adjoint to G [prehension]
 - if we can define [satisfaction]
 - Unit of adjunction $\eta: L \rightarrow GFL$
 - Counit of adjunction $\varepsilon: FGR \rightarrow R$
 - Commuting diagrams involving η, ε, F, G
 - » Where L, R are objects in \mathbf{L}, \mathbf{R} respectively
 - Whole is a snap, a grasping, not a sequence
- F is a free functor (creativity) with change η [concrecence]
- G is an underlying functor (applies rules) with change ε [obligation]
- Special case
 - No change in η or ε then equivalence relationship between F and G

Adjointness -- Motivation

- Adjointness between functors provides a formal basis for relationships which for applied category theory
 - escapes the clutches of Gödel's undecidability to provide a metaphysical approach to higher-order logic.
 - enables relationships to be specified that are 'less than' equivalence
 - but which are common in real world
 - e.g. language translation
 - is natural with respect to composition

Category of the Ultimate PR p.21

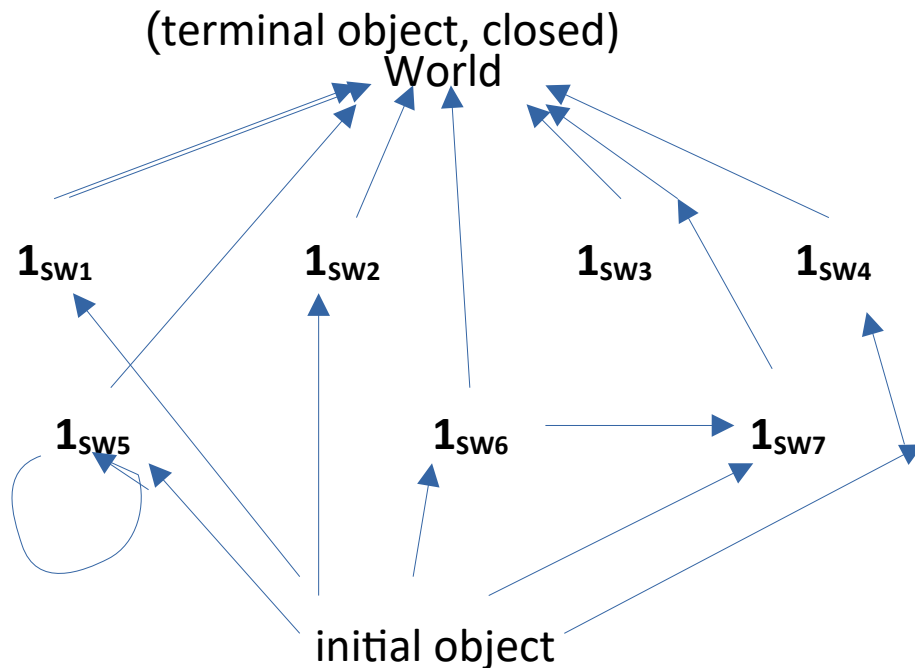
“ 'Creativity' 'many' 'one' are the ultimate notions involved in the meaning of the synonymous terms 'thing' 'being' 'entity'. These three notions complete the Category of the Ultimate and are presupposed in all the more special categories.

The term 'one' does not stand for 'the integral number *one*' which is a complex special notion. It stands for the general idea underlying alike the indefinite article '*a* or *an*' and the definite article '*the*' and the demonstratives '*this* or *that*' and the relatives '*which* or *what* or *how*.' It stands for the singularity of an entity. The term 'many' presupposes the term 'one' and the term 'one' presupposes the term 'many'. The term 'many' conveys the notion of 'disjunctive diversity'; this notion is an essential* element in the concept of 'being'. There are many 'beings' in disjunctive diversity. ”

- In many respects this is a topos.

Topos: one and many

- World Category $\mathbf{1}_c$ with Categories $\mathbf{1}_{sw}$ subworlds



Identity arrow for preorder \mathbf{C} :
 $\mathbf{1}_c: \mathbf{C} \rightarrow \mathbf{C}$

$\mathbf{1}_c$ is the identity functor

Provides a handle (the one) for the world and its subworlds

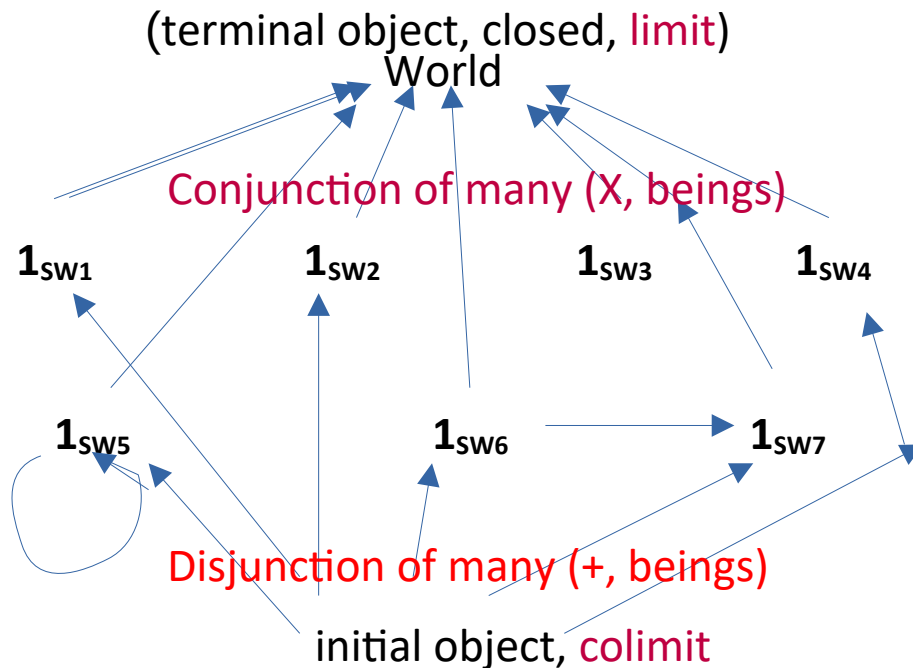
Category of the Ultimate PR p.21 (continued)

“ 'Creativity' is the universal of universals characterizing ultimate matter of fact. It is that ultimate principle by which the many, which are the* universe disjunctively, become the one actual occasion, which is the universe conjunctively. It lies in the nature of things that the many enter into complex unity.”

- The topos 1_c

Topos: Disjunction (colimit) and Conjunction (limit)

- World Category $\mathbf{1}_c$ with Categories $\mathbf{1}_{sw}$ subworlds



Identity arrow for preorder \mathbf{C} :
 $\mathbf{1}_c: \mathbf{C} \rightarrow \mathbf{C}$

$\mathbf{1}_c$ is the identity functor

Provides a handle (the one) for the world and its subworlds

Category of the Ultimate PR p.21 (continued)

“ The ultimate metaphysical principle is the advance from disjunction to conjunction, creating a novel entity other than the entities given in disjunction. The novel entity is at once the togetherness of the 'many' which it finds, and also it is one among the disjunctive 'many' which it leaves; it is a novel entity, disjunctively among the many entities which it synthesizes. The many become one, and are increased by one. In their natures, entities are disjunctively 'many' in process of passage into conjunctive unity. This Category of the Ultimate replaces Aristotle's category of 'primary substance' “.

- Again the topos 1_c
 - Tension between X (conjunction) and + (disjunction) featured strongly in our last ANPA paper on music
- Aristotle's category of 'primary substance' is extensional
 - His secondary substance is intensional
 - Intension is an inherent part of each category through Dolittle diagrams (see our last ANPA paper on music)

There are eight Categories of Existence PR p.22

- “ (i) Actual Entities (also termed Actual Occasions), or Final Realities, or Res Verae.
- (ii) Prehensions, or Concrete Facts of Relatedness.
- (iii) Nexus (plural of Nexus), or Public Matters of Fact. Later p.34 nexus is a society with social order.
- (iv) Subjective Forms, or Private Matters of Fact.
- (v) Eternal Objects, or Pure Potentials for the Specific Determination of Fact, or Forms of Definiteness.
- (vi) Propositions, or Matters of Fact in Potential [33] Determination, or Impure Potentials for the Specific Determination of Matters of Fact, or Theories.
- (vii) Multiplicities, or Pure Disjunctions of Diverse Entities.
- (viii) Contrasts, or Modes of Synthesis of Entities in one Prehension, or Patterned Entities.

Among these eight categories of existence, actual entities and eternal objects stand out with a certain extreme finality. The other types of existence have a certain intermediate character. “

The Eight Categories of Existence in Category Theory

- (i) Actual Entities [building blocks for categories]
- (ii) Prehensions, Relatedness [adjointness]
- (iii) Nexus (society) [union of entities with order]
- (iv) Subjective Forms [pullback, pushout]
- (v) Eternal Objects [constants]
- (vi) Propositions [logic]
- (vii) Multiplicities, or Pure Disjunctions [colimit]
- (viii) Contrasts, or Modes of Synthesis [adjointness]

The Categories of Explanation PR p.22

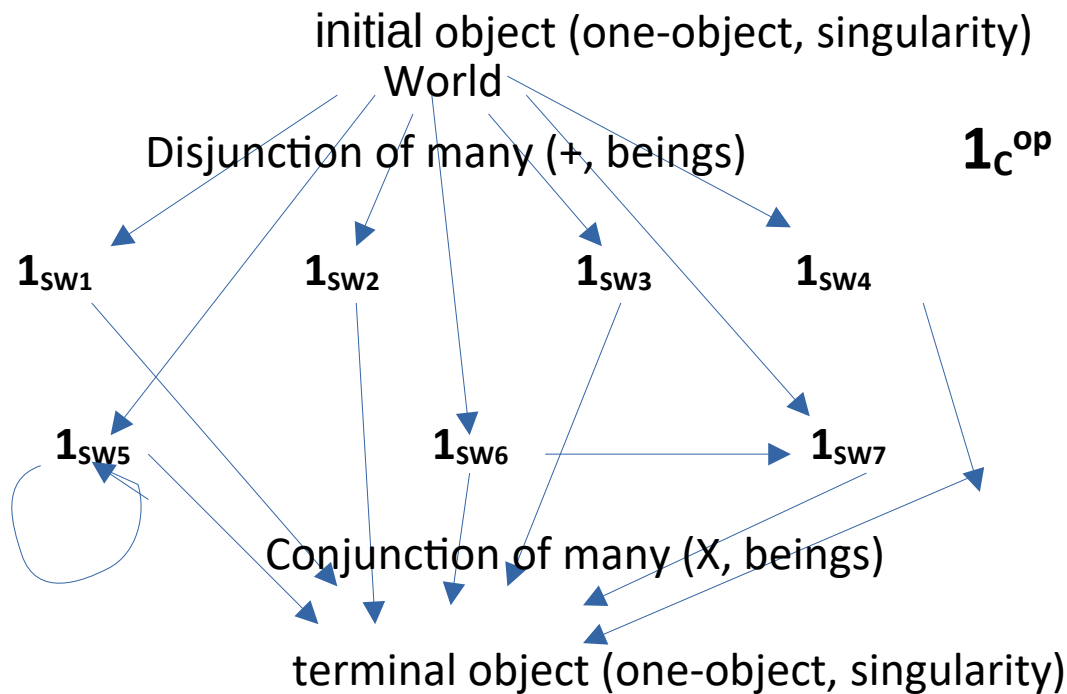
“ There are twenty-seven Categories of Explanation:

(i) That the actual world is a process, and that the process is the becoming of actual entities. Thus actual entities are creatures; they are also termed 'actual occasions' “

- Becoming is the creative process
- Occasions are Now (in time)

Addressing through Category $\mathbf{1}_C^{OP}$

- Inverse (dual) of World Category $\mathbf{1}_C$ with Categories $\mathbf{1}_{SW}$ subworlds



The inverse of each arrow compose together into a 'Now' free functor of instantaneous World Events.

Now is after Whitehead, an actual occasion

Logic is Heyting, intuitionistic

Creativity

Becoming (process): New actual entity $1_C^{OP'}$ From 1_C^{OP}

Actual occasion: in incremental time

Free functor, concrescence

$$F: \mathbf{1}_C^{op} \longrightarrow \mathbf{1}_C^{op'}$$

Underlying functor, obligation

$$G: \mathbf{1}_C^{op'} \longrightarrow \mathbf{1}_C^{op}$$

Adjointness, prehension

$$F \dashv G \quad \langle F, G, \eta, \varepsilon \rangle$$

Unit of adjunction, satisfaction

$$\eta: \mathbf{1}_C^{op} \longrightarrow GF \mathbf{1}_C^{op}$$

Counit of adjunction, satisfaction

$$\varepsilon: FG \mathbf{1}_C^{op'} \longrightarrow \mathbf{1}_C^{op'}$$

The Categories of Explanation PR p.22

“ (iv) That the potentiality for being an element in a real concrescence* of many entities into one actuality is the one general metaphysical character attaching to all entities, actual and non-actual; and that every item in its universe is involved in each concrescence. In other words, it belongs to the nature of a 'being' that it is a potential for every 'becoming'. This is the 'principle of relativity'. “

- That being is a potential for becoming

The Categories of Explanation PR pp.23-24

“ (xii) That there are two species of prehensions: (a) 'positive prehensions' which are termed 'feelings,' and (b) 'negative prehensions' which are said to 'eliminate from feeling.' Negative prehensions also have subjective forms. A negative prehension holds its datum as inoperative in the progressive concrescence of prehensions constituting the unity of the subject,

(xiii) That there are many species of subjective forms, such as emotions, valuations, purposes, aversions, aversions, consciousness, etc. “

- That positive prehensions include feelings, while negative prehensions do not include feelings.
- That subjective forms (of relationships) include emotions

The Categories of Explanation PR p.24

- “ (xviii) That every condition to which the process of becoming conforms in any particular instance has its reason either in the character of some actual entity in the actual world of that concrescence, or in the character of the subject which is in process of concrescence. This category of explanation is termed the 'ontological principle.' It could also be termed the 'principle of efficient, [37] and final, causation' This ontological principle means that actual entities are the only reasons; so that to search for a reason is to search for one or more actual entities. It follows that any condition to be satisfied by one actual entity in its process expresses a fact either about the 'real internal constitutions' of some other actual entities, or about the 'subjective aim' conditioning that process. “
- Ontology here deals with the explanation in terms of reasons as to how becoming (the process) is derived from being (the actual entities).
 - Mapping between categories (actual entities) and adjointness (prehension)

The Categories of Explanation PR pp.25-26

“ (xxv) The final phase in the process of concrescence, constituting an actual entity, is one complex, fully determinate feeling. This final phase is termed the 'satisfaction'. It is fully determinate (a) as to its genesis, (b) as to its objective character for the transcendent creativity, and (c) as to its prehension—positive or negative—of every item in its universe.”

- That concrescence is a process in which prehensions are integrated into a fully determinate feeling or satisfaction.
 - Satisfaction and feelings are the closure of the prehension (and of the adjointness)

There are nine Categorical Obligations PR pp.26-27

“ (i) The Category of Subjective Unity ...

(ii) The Category of Objective Identity ...

(iii) The Category of Objective Diversity ...

(iv) The Category of Conceptual Valuation ...

(v) The Category of Conceptual Reversion ...

(vi) The Category of Transmutation ...

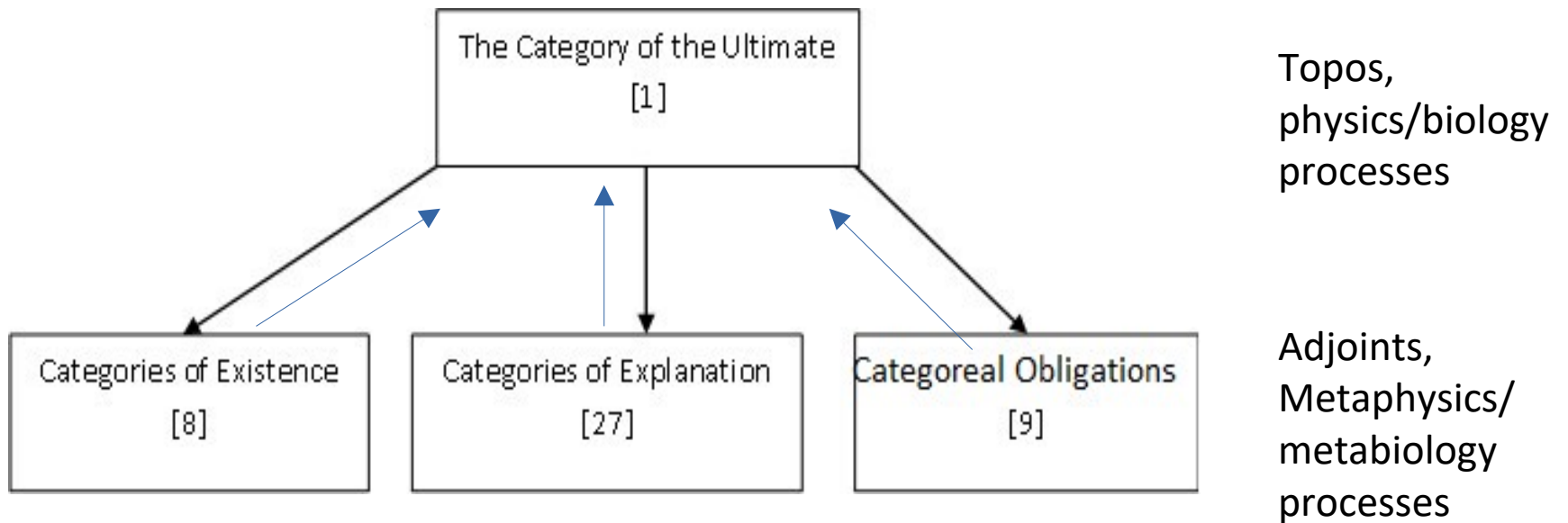
(vii) The Category of Subjective Harmony ...

(viii) The Category of Subjective Intensity ...

(ix) The Category of Freedom and Determination ... “

- Concerned with maintenance of integrity
 - Underlying functor in adjointness
 - Modal logic

Overall Scheme Revisited



Numbers are count of categories of each type

Not a hierarchy but an adjointness

Results 1

- The Category of the Ultimate is a topos, handling the process ‘becoming’
 - In a single-substance philosophy
 - Mind and matter are treated as one
 - The match between Whitehead’s language and topos theory is very close
- The other special categories (Existence, Explanation, Obligations) expand and control ‘becoming’:
 - Can be viewed as metaphysics/metabiology

Results 2

- Whitehead makes extensive use of some terms of his own, which we have related to category theory:
 - ‘Prehension, relatedness’ – adjointness
 - ‘Nexus, togetherness’ – union of entities of same type with order
 - ‘Concrescence’ – free functor
 - ‘Obligations’ – underlying functor
 - ‘Satisfaction’ – adjoint conditions realised
 - ‘Subjective Forms’ – pullback, pushout
- Our interpretations could be challenged

Results 3

- Categorification
 - Simple-minded translation 1:1 of concepts into category theory
- Have tried to avoid, looking at main thrust of Whitehead's work
 - Not sensible to translate someone's ideas expressed in great detail in words into my formalism
 - Not possible to understand all of Whitehead's text as he writes in an introspective way
- Still looking at later sections, with more explanation

Consequences

- Whitehead's Process and Reality provides a philosophical basis for category theory
- This is an alternative to the classical pure mathematical/set-based basis
- Process and Reality gives a much richer realism than set theory, facilitating
 - process as a single substance, combining mind and matter
 - a much richer type system, including complex metadata in the three additional special categories
 - a move towards biology
- A pure mathematician would say 'What are the types?'