A Formal Basis for Metabiology

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ANPA, Liverpool University, August 2023

Motivation

- Biology has always been the poor relation of physics when it comes to mathematical constructions.

- Metabiology appears to have no clear definition and no pedigree, unlike metaphysics which has a connection to Aristotle.

- This is partly due to the perceived difficulty of describing the complex organisms and their transitions that underpin biology.

- But another important factor has been the lack of tools at the appropriate conceptual level.

Prior Attempts at Defining Metabiology

- Arturo Carsetti discusses metabiology in terms of:
  - Husserls' phenomenology, sensory intuition and categorial intuition
  - Gödel's sentences
  - Lawvere's Cartesian closed categories
  - Halmos' self-organising/intensional net with polyadic algebras
  - Atlan's higher-order cybernetics (self-organizing).
- Chaitin coined the term metabiology in attempting to prove Darwinian theory through algorithms.
- Turing developed wavelike patterns for biology that are the chemical basis for morphogenesis.

Other Work on Metabiology

- Robert Rosen: Life Itself
  - Relational biology
  - Uses categorical functors to describe living systems
- Whitehead: Process and Reality
  - Prehension
  - Concrescence
  - Eternal Objects

What do we mean by Meta?

- 'after', 'behind', 'beyond' [Wiki, metaphysics]

- 'data that provides information about other data' [Wiki, metadata]

- 'the study of mathematics itself using mathematical methods' [Wiki, metamathematics]

- 'in the middle, between' [Wiki, organic chemistry, benzene ring, meta substitution]

ortho

meta

para
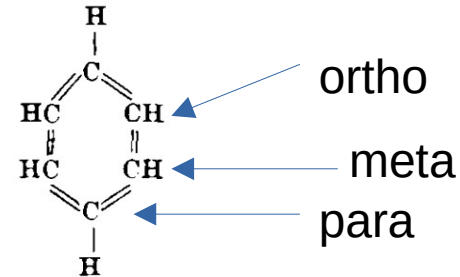
Metaphysics

- Not a term coined by Aristotle
- Used by successors to describe a body of texts
  - to be studied after the ones dealing with nature
  - First philosophy
    - Causes and principles
    - Being, existence
- More general and abstract principles, wisdom
  - After the physics

Metamathematics

- Proponents Frege, Hilbert, Kleene.
- Looking for foundation to mathematics
    - Generalising proofs
    - Metatheories
- Gödel's incompleteness theorems showed limitations in proofs on axiomatic systems
- Been superseded by mathematical logic to some extent

Metadata

- Well established in computing science

- Mapping specific data definitions to general principles of data structuring

- Essential for interconnection of systems (interoperability)

- Can have multiple levels of abstraction (metameta is better-better!)

Usage of meta in metabiology

- Should attempt to build a theory on general principles
  - Abstracting from the specific
- Should be based on:
  - General processes
    - Applied to abstract relations
  - Clear logical principles

Relevant for Biology 1

- Organism
  - An organism is any biological living system that functions as an individual life form. All organisms are composed of cells (cell theory)
  - Could be a single cell or a composition of cells
  - Any individual animal, plant, bacterium, etc. having various parts or systems that function together as a whole to maintain life and its activities
  - a form of life composed of mutually interdependent parts that maintain various vital processes

Relevant for Biology 2

- Processes
  - Movement, reproduction, sensitivity, nutrition, excretion, respiration and growth
  - Birth and death
  - Evolution reflects the adaptations of organisms to their changing environments and can result in altered genes, novel traits, and new species.
  - Anticipation
    - natural system's causal entailment
    - model will be able to accurately predict future behaviours of the original, natural system.

Mathematical Properties

- Identity – assured as organisms are individual
- Internal structure – not modelled as organisms can be considered to be encapsulated (DNA?)
- Process is therefore more important than structure
- Essentially categorial with the arrow the primary modelling construct

Two Strong Rival Candidates for Metabiology

- Robert Rosen
  - Fundamental look at biological systems in terms of *Life Itself* (his book) and Anticipatory Systems
  - Uses formal categorial functors

- Alfred Whitehead
  - In Process & Reality develops for organisms a philosophy for *being* and *becoming*, the latter through concrescence
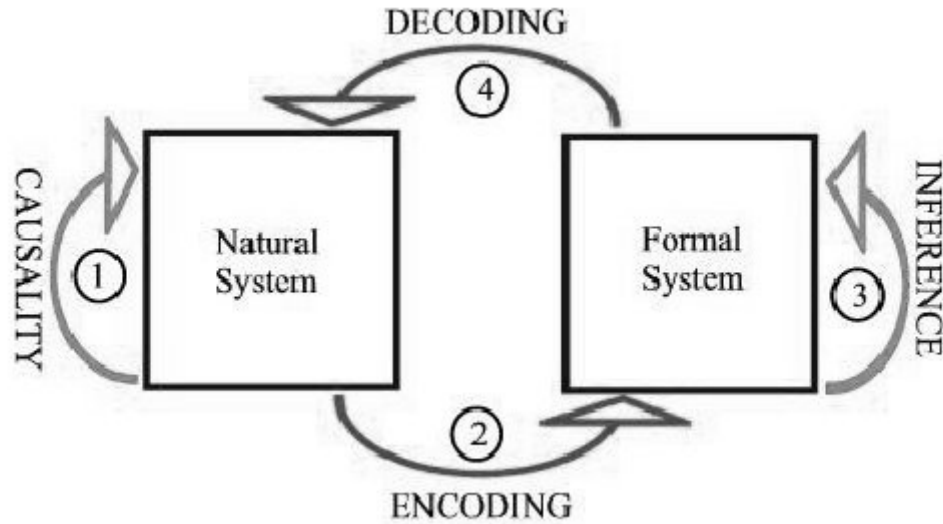 - Uses informal category theory

Robert Rosen (1934-1998)

- Prominent theorist in the areas of biology and biophysics at Buffalo and Dalhousie

- Influenced by Rashevsky's On Growth and Form (1917) - Biotopology

- Believed in complex systems

  – Against reductionism/reconstruction

- Did not subscribe to Descartes' idea that Animals are efficient machines

- Complexity refers to the causal impact of organization on the system as a whole

- Rosen's presentation of his modelling framework, (M, R) systems, is highly abstract (M Metabolism, R Repair)

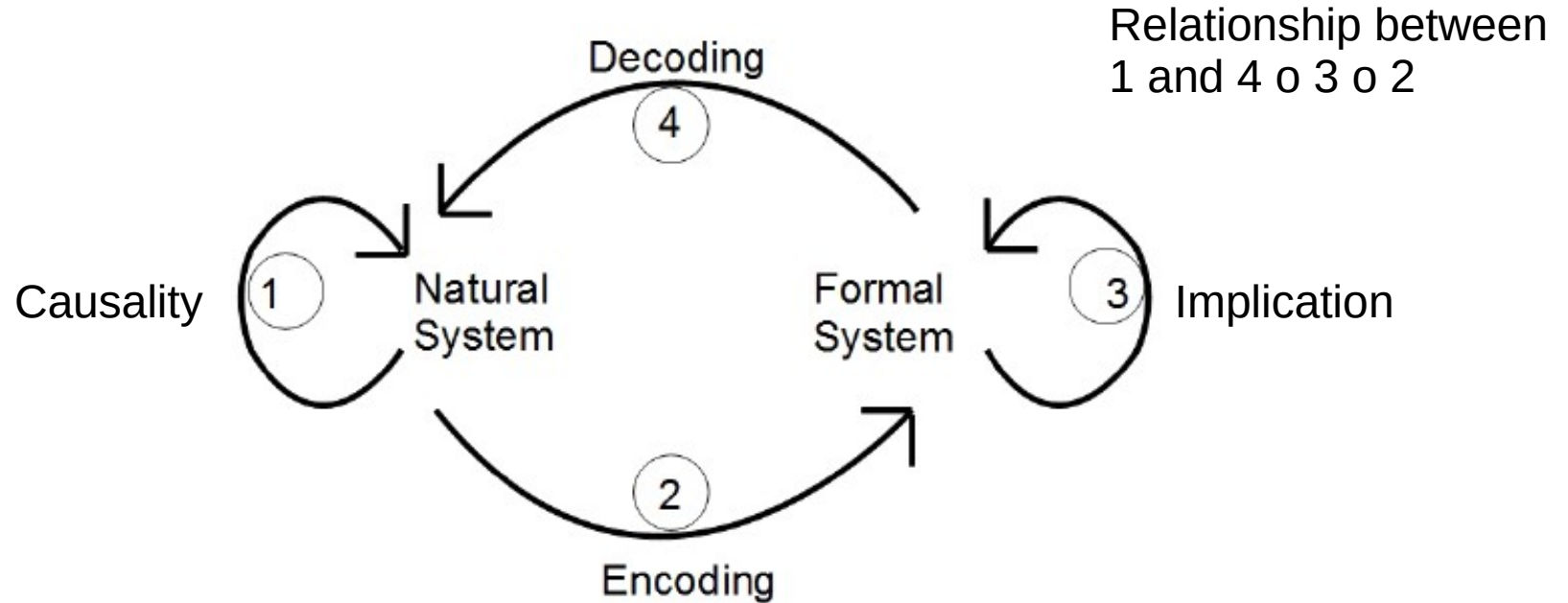- Rosen gives few working examples of biological systems

Rosen Publications

- 1985, Anticipatory Systems: Philosophical, Mathematical and Methodological Foundations. Pergamon Press. Reprinted 2005 by Columbia Press.

- 1991, Life Itself: A Comprehensive Inquiry into the Nature, Origin, and Fabrication of Life, Columbia University Press

- Published posthumously:

- 2000, Essays on Life Itself, Columbia University Press.

- 2012, Anticipatory Systems; Philosophical, Mathematical, and Methodological Foundations, 2nd Edition, Springer

Rosen's Modelling Relation

1991 diagram 7F.1

Robert Rosen – Our Adaption

Relationship between 1 and 4 o 3 o 2

Decoding

Causality

Natural System

Formal System

Implication

Encoding

Modelling a Natural System with a Formal System

Rosen's Functors

- Causality – cause and effect – ①
- Encoding – representation in model (free functor) – ②

- Implication – inference – ③
- Decoding – verification of model (underlying functor) - ④

Commuting Problems

- Rosen had difficulties in his earlier work in showing how the hybrid diagram of biological and mathematical equivalences commutes
    - Does ① = ④ o ③ o ②?
    - With category theory: If equivalent yes. May also be adjoint.
- Rosen offered a formal translation using the block diagram approach of the general and logical theory of automata of McCulloch, Pitts, and von Neumann.
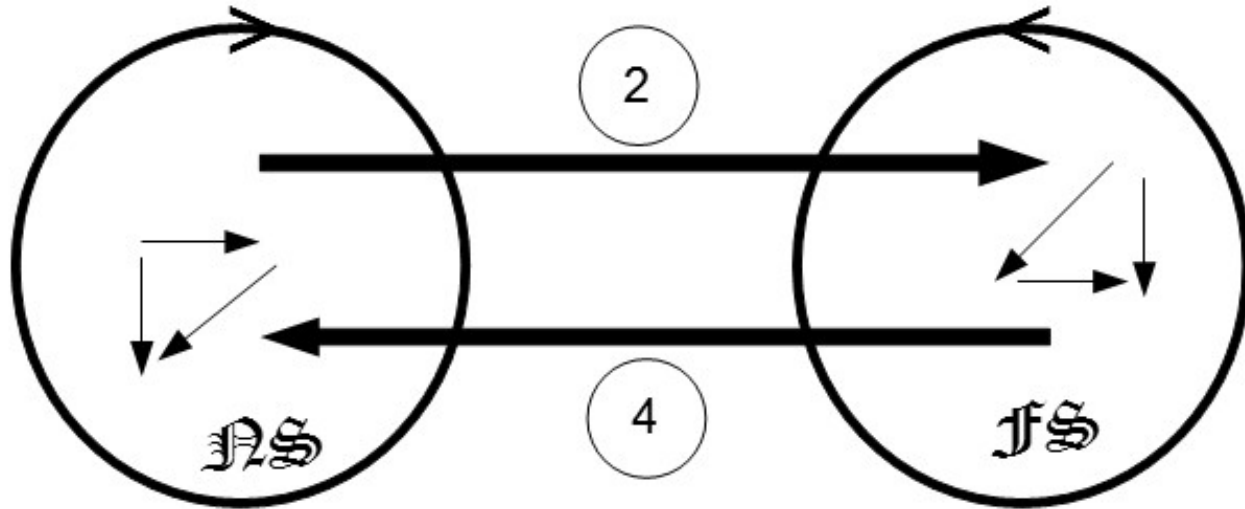
Relational Holon based on Rosen's modelling relation

Bidirectional arrows between encoding and Decoding are introduced
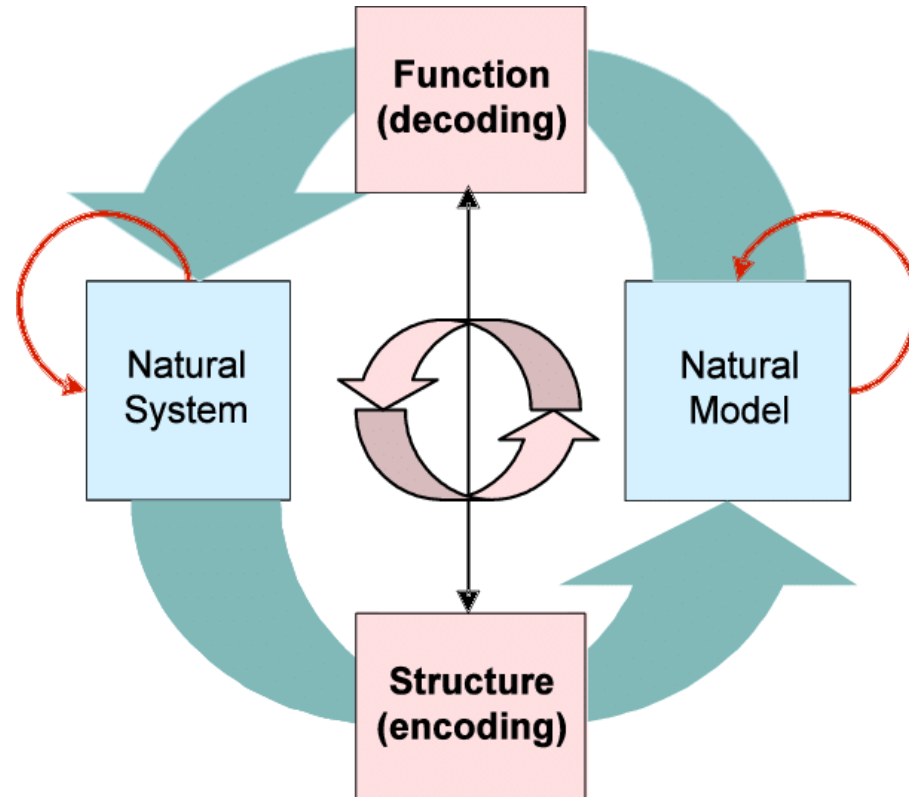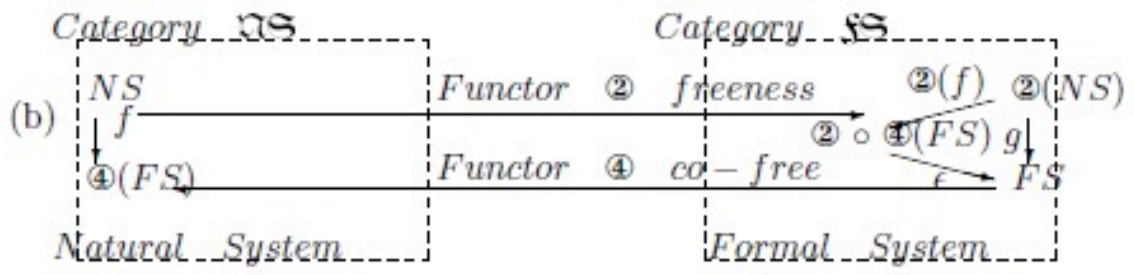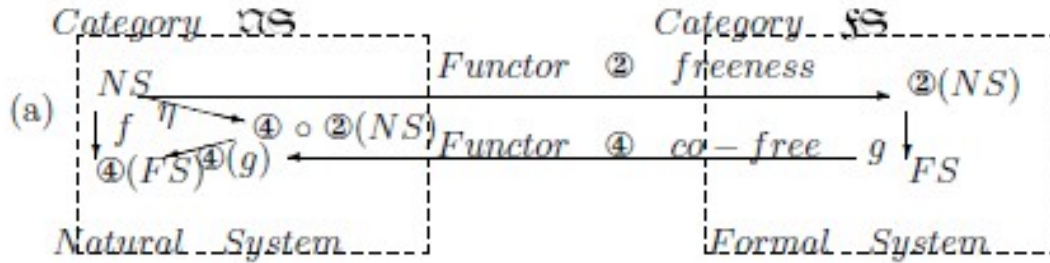

John J Kineman
Relational Theory and Ecological Niche Modelling

Heather & Rossiter Liège 2009

Two-way Mapping of Functors ② and ④ between categories NS and FS (Gothic as large categories)
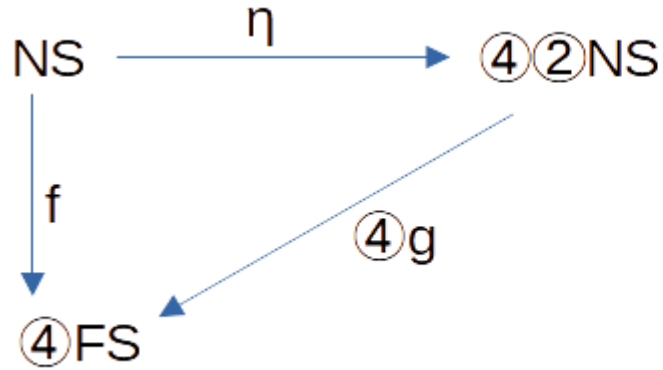
Details of Adjointness

②⊣④ <②, ④, η, ε>
② is left adjoint to ④, ④ is right adjoint to ②,
4-tuple holds functors ②, ④, η as unit of adjunction, ε as counit of adjunction

Unit, Counit of Adjunction

Unit of adjunction
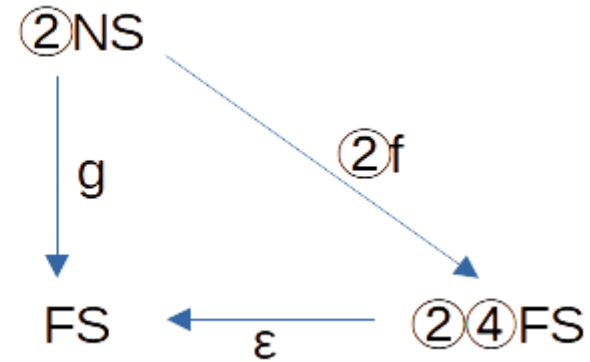
Counit of adjunction

Rosen's Potential

- Fundamental Biological Processes modelled

- Enables comparison of biological causality with formal inferential modelling

  - Modelling can be anticipatory

- Ideally inferences in model and natural causality give equivalence

- Adjointness widens scope of model to handle perturbations

Whitehead

- In Process & Reality 1928
    - Develops the idea of feelings
        - Based upon concrescence
            - of prehensions from
                - Ingression of eternal objects/actual entities

- A pithy summary

Broader View

- Feelings are sentient, not necessarily conscious.
- Prehension (being) does not necessarily lead to a concrete new entity (becoming, via concrescence)
  - Emergence
  - Evolution (Darwin)

Entities

- Real (exist)

- Individual (atomic)

- Particular (singled out, identity)

- Can be joined together as a nexus (union of similar entities, ordered society)

- "Every entity should be a specific instance of one category of existence" PR p.20 (classification)

- Can be seized by prehension (product or coproduct relatedness) to give a subjective form

Eternal Objects are Atemporal

- Any entity whose conceptual recognition does not involve a necessary reference to any definite actual entities of the temporal world is called an `eternal object'.

- Eternal objects are therefore time-invariant.

- "xv. That a proposition is the unity of certain actual entities in their potentiality for forming a nexus, with its potential relatedness partially defined by certain eternal objects which have the unity of one complex eternal object."

Examples of Eternal Objects

- Eternal objects are potentials
  - Could be an object like the number 2
  - But also include sensory qualities, like colours and tactile sensations; conceptual abstractions like shapes; numbers; moral qualities; physical fundamentals; feelings like an emotion, adversion, aversion, pleasure or pain; qualia
  - Artistic performances, a scientific paper
- Giving conceptual prehension in form of feelings

Prehension – Concrete Definition PR p.23

"(xi) That every prehension consists of three factors: (a) the 'subject' which is prehending, namely, the actual entity in which that prehension is a concrete element; (b) the 'datum' which is prehended; (c) the 'subjective form' which is how that subject prehends that datum."

- This is a data relationship, either X or +.

- A pullback (or pushout) category:

   – (c) is (a) $X_{(b)}D$ (D is a third entity)

   – (c) is the subjective form

- Cartesian closed (X) (or Cocartesian closed (+)) category

# Category CPB: the Limit Diagram of a Category C: the Pullback A X_B D
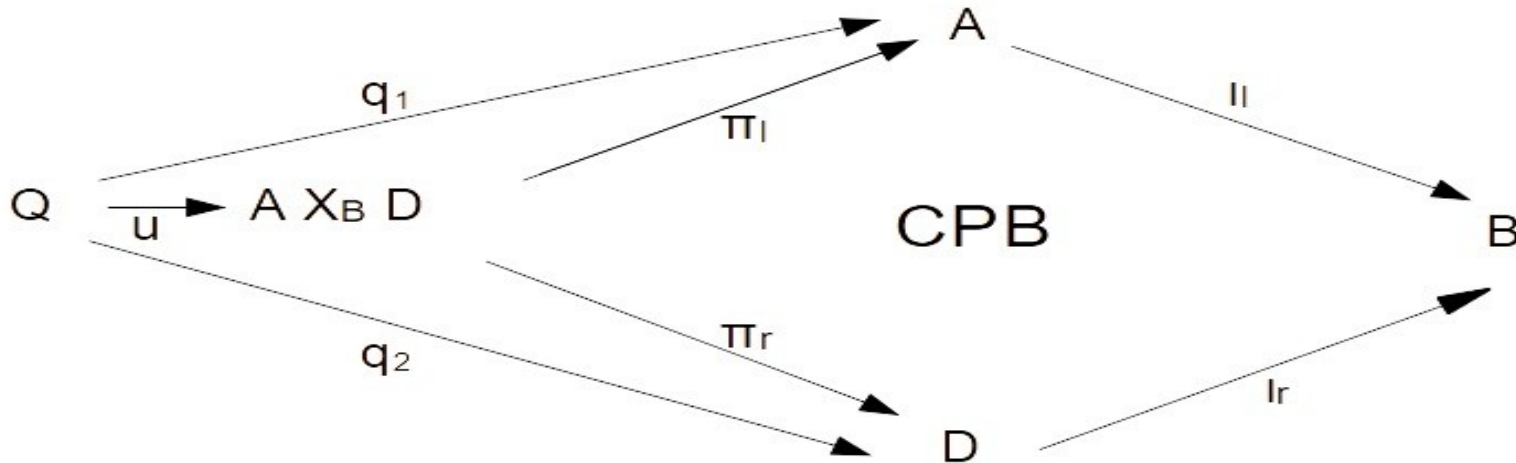
The subjective form of Whitehead is the product A X_B D

A = (a), B = (b), (c) = A X_B D, D is introduced

Concrescence

• Prehension is a process by which an actual entity, or prehending subject, becomes itself by appropriating elements from other actual entities.
• The becoming of an actual entity occurs through a concrescence of prehensions.
• Satisfaction is a final phase of concrescence (or the process of integration of feeling), in which prehensions are integrated into a concrete unity.

The locally Cartesian closed category LCCC: Adjointness in the functors between the product A ×$_B$ D and B
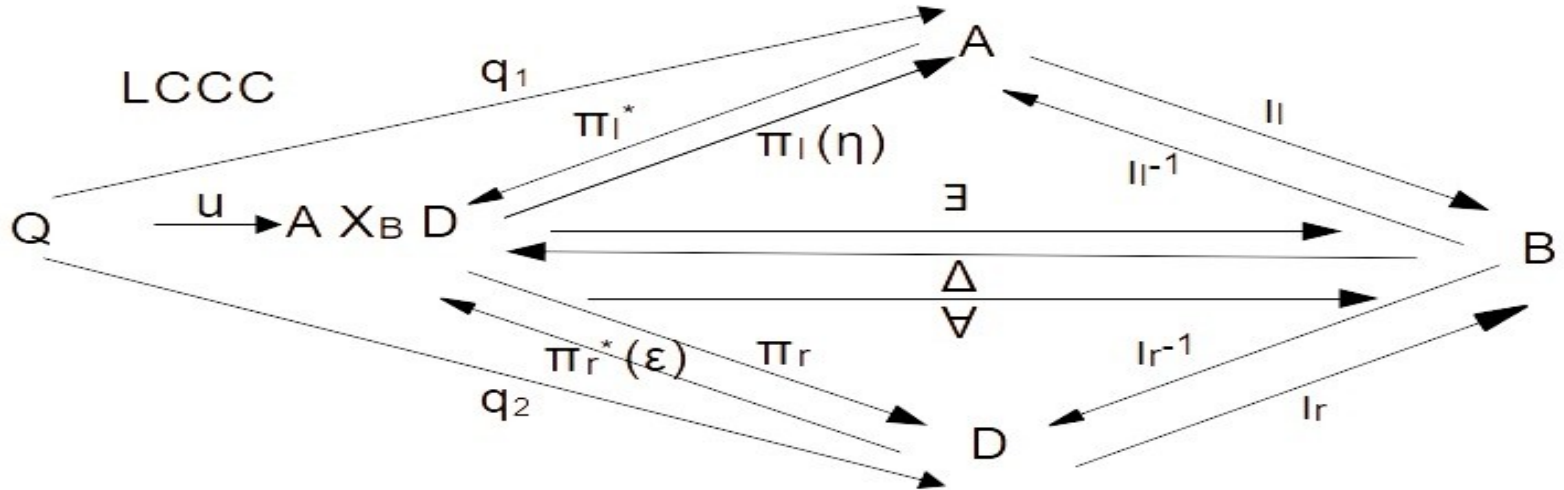
The hyperdoctrine of Lawvere (1969): $\exists \dashv \Delta \dashv \forall$
Adjointness

Adjointness gives Viability

Whitehead:

- Prehension gives relatedness (being)

- Concrescence gives viability of relations (becoming)

Categories:

- Pullbacks give relations

- Adjointness gives viability

Nexus – Defining Characteristic

- A nexus enjoys `social order' where (i) there is a common element of form illustrated in the definiteness of each of its included actual entities, and (ii) this common element of form arises in each member of the nexus by reason of the conditions imposed upon it by its prehensions of some other members of the nexus, and (iii) these prehensions impose that condition of reproduction by reason of their inclusion of positive feelings of that common form.

- Such a nexus is called a `society' and the common form is the `defining characteristic' of the society. The notion of `defining characteristic' is allied to the Aristotelian notion of `substantial form'.

- Thus the nexus forms a single line of inheritance of its defining characteristic.

The Colimit Diagram of Category NEX: the Pushout DC $+_{INH}$ S
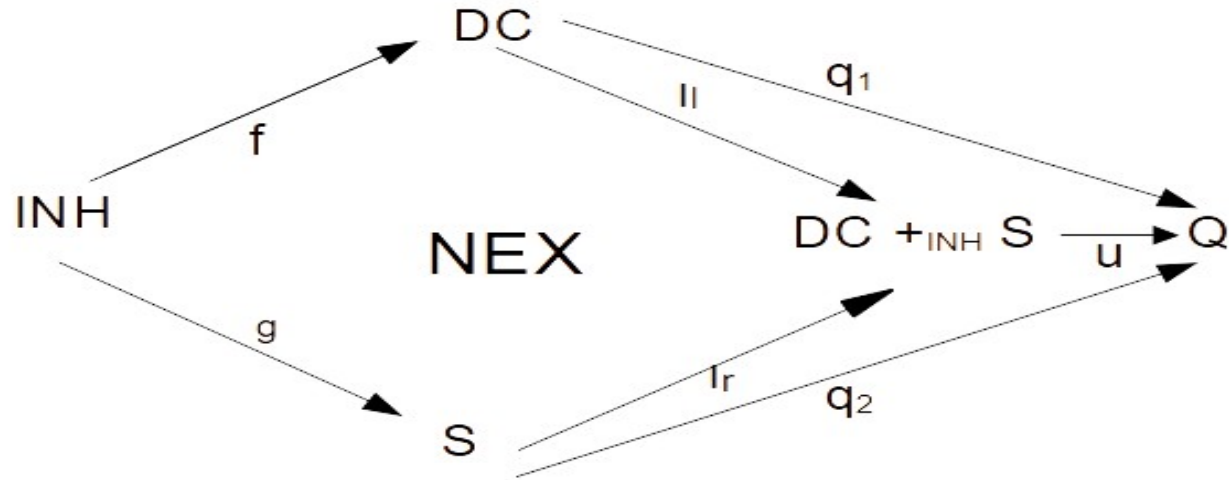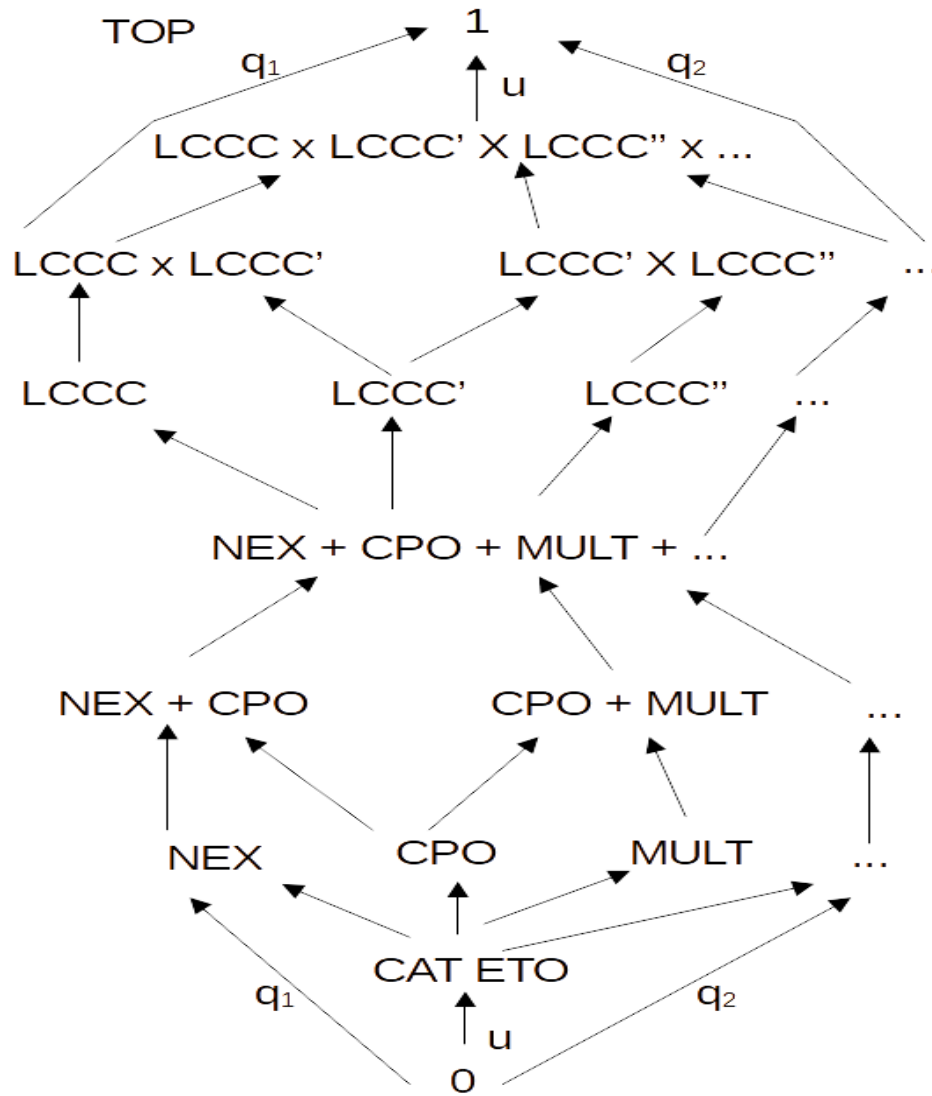

S = Society, DC = Defining Characteristic, INH = line of inheritance

Category of the Ultimate PR p.21

" The ultimate metaphysical principle is the advance from disjunction to conjunction, creating a novel entity other than the entities given in disjunction. The novel entity is at once the togetherness of the 'many' which it finds, and also it is one among the disjunctive 'many' which it leaves; it is a novel entity, disjunctively among the many entities which it synthesizes. The many become one, and are increased by one. In their natures, entities are disjunctively 'many' in process of passage into conjunctive unity. This Category of the Ultimate replaces Aristotle's category of 'primary substance' ".

- Tension between X (conjunction) and + (disjunction) featured strongly in our last ANPA paper on music

- Aristotle's category of 'primary substance' is extensional

  – His secondary substance is intensional

  – Intension is an inherent part of each category through Dolittle diagrams (see our last ANPA paper on music)

The Topos
Category TOP:
Tension between
times (prehension)
and *plus* (nexus)

TOP

1

q_1 q_2

u

LCCC x LCCC' X LCCC" x …

LCCC x LCCC'   LCCC' X LCCC"   …

LCCC   LCCC'   LCCC"   …

NEX + CPO + MULT + …

NEX + CPO   CPO + MULT   …

NEX   CPO   MULT   …

CAT ETO

q_1 q_2

u

0

Terminal object
Limit cone
One (singular)

Conjunctions
Many

Prehension
(pullback) with
concrescence

Disjunctions
Many

Nexus
Multiplicities
Prehension
(pushout)

Actual Entities
Eternal objects
Many

Initial object
Colimit cone

Results 1

- The Category of the Ultimate is a topos, handling the process 'becoming'
  - In a single-substance philosophy
    - Mind and matter are treated as one
  - The match between Whitehead's language and topos theory is very close
- The other special categories (Existence, Explanation, Obligations) expand and control 'becoming':
  - Can be viewed as metaphysics/metabiology

Results 2

- Whitehead makes extensive use of some terms of his own, which we have related to category theory:

    – 'Prehension, relatedness' – Cartesian/Cocartesian closed category
    – 'Subjective Forms' – pullback, pushout
    – 'Nexus, togetherness' – Cocartesian closed
    – 'Concrescence' – adjointness (Locally Cartesian closed category after Lawvere)
    – 'Obligations' – underlying functor (in adjointness}
    – 'Satisfaction' – adjoint conditions realised
    – 'Category of the Ultimate' – topos (Lawvere's hyperdoctrine)

Whitehead's Potential

- Emergence and evolution through prehension and concrescence

- What about causality?

- Prehensions can be nested, with a subjective form from one prehension becoming the object (datum) of another prehension

  - Giving subject-object pairs

  - Part of Topos diagram earlier showing nested conjunctives

Rosen vs Whitehead

- From biology viewpoint
  - Both handle causality
  - Whitehead additionally handles evolution
  - Rosen additionally handles anticipation
- From formal viewpoint
  - Both are amenable to category theory formalisation
- From meta viewpoint
  - Rosen has a simpler mechanism including model handling
  - Whitehead's principles are not always explicit
- From organism structure viewpoint
  - Both are process orientated
- A score draw, at present