Whitehead's Category of the Ultimate as Lawvere's Topos in Category Theory

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Category theory has its foundations in the pure mathematics of objects related by arrows. Many of the proofs in category theory are on small categories, set-like constructions. The philosopher Alfred North Whitehead developed an alternative basis for categories in his book Process & Reality in the description of the Category of the Ultimate. Whitehead developed an intricate textual description of a number of categories, linked to the Category of the Ultimate, including eight Categories of Existence, nine Categoreal Obligations and 27 Categories of Explanation. Whitehead introduces terms such as being for existence, becoming for transformation by prehension with concrescence, nexus for a society and propositions for logic in the broader sense.

Whitehead's Categories can be considered in terms of mathematical categories as products (times) or union (plus), such as in prehension or nexus. Concrescence is adjointness between times and plus, giving the closure to create a locally Cartesian closed category as a becoming. The Category of the Ultimate is creation from the singular colimit through many disjunctive collections (plus) and their many conjunctions (times) to the singular limit. This paper argues that the Category of the Ultimate corresponds closely to a topos as in Lawvere's work in category theory in 1969 with the progression from colimit (plus) to limit (times) in the hyperdoctrine.

When Whitehead published his book in 1929, category theory did not exist. It was not until 1945 that Eilenberg and MacLane produced the first treatise on category theory. So the developments are independent of each other. Like category theory, based solely on the arrow, Whitehead's approach is single substance with no mind-matter separation as in the work of Descartes. Whitehead's focus though is much broader, extending beyond the simpler physical constructions and limited standard types of category theory. For example, eternal objects, including the senses, are available for ingression by actual entities to give conceptual prehensions, yielding feelings, which can be used to express metaphysical, metabiological and social concepts, with their complex types. Whitehead does have empathy with Aristotle: his Category of the Ultimate is of Aristotle's first substance, that is extensional, and his Defining Characteristic in the Nexus is of Aristotle's second substance, that is intensional.

Building on the arrow of mathematical category theory, Whitehead's work does therefore deserve to be treated as a philosophical basis for a more adventurous type of category theory with its additional emphasis on processes in general, including social and biological ones. An extensible type system for feelings, based on AI and biological concepts, will be required.